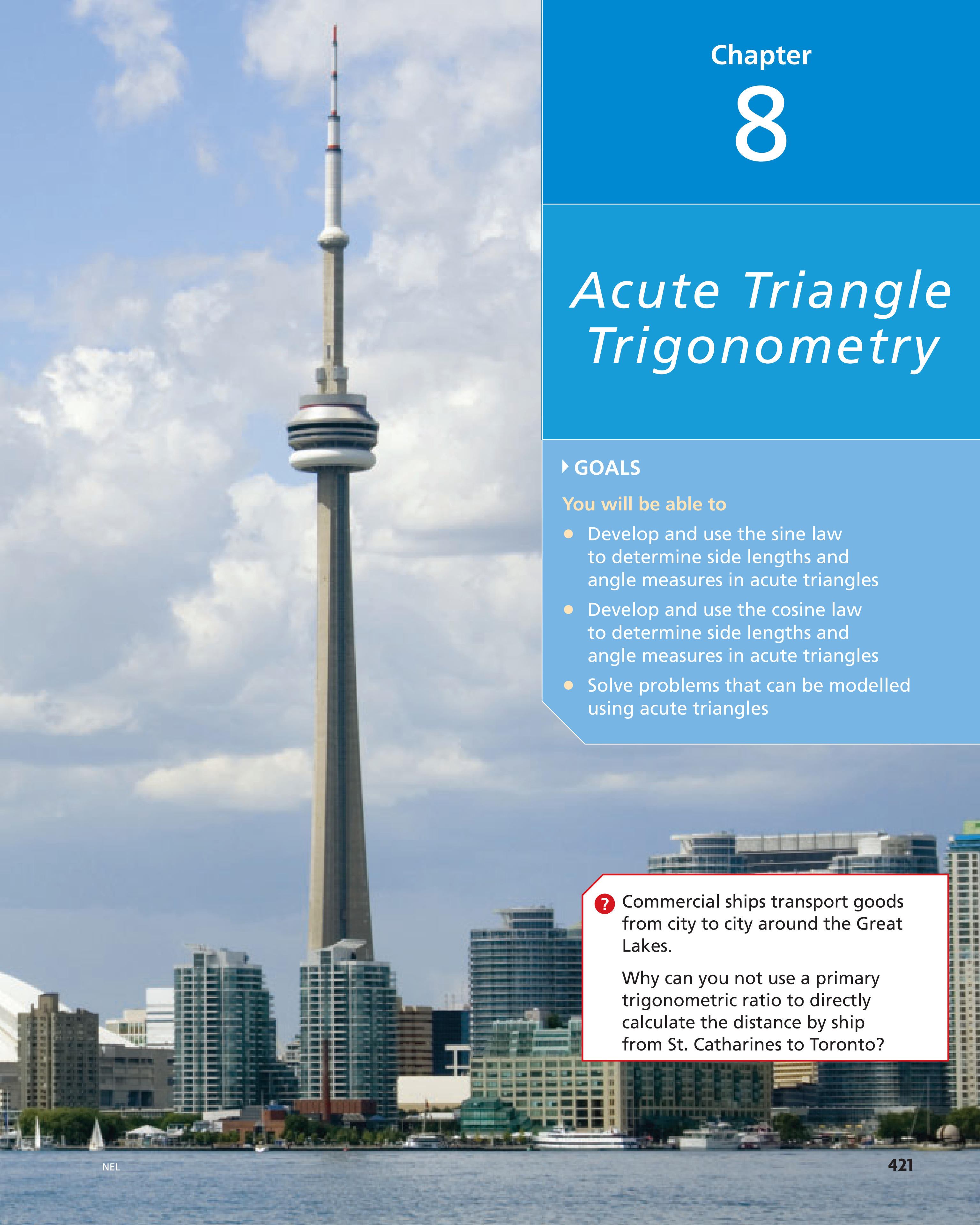

Aurora
Whitchurch-Stouffville
Orangeville
Caledon
Bolton
Richmond Hill
Markham
Pickering
Ajax
Vaughan
Brampton
Georgetown
TORONTO
Mississauga
Lake Ontario
Guelph
Milton
Oakville
Cambridge
Burlington
Dundas
Hamilton
Stony Creek
Grimsby
St. Catharines
Niagara-on-the-Lake
Niagara Falls
U.S.A.
Brantford
9
400
404
407
25
7
10
11
401
24
6
403
8
5
QEW
406
20
54 km
43 km
63°
?

NEL



Chapter 8

Acute Triangle Trigonometry

GOALS

You will be able to

- Develop and use the sine law to determine side lengths and angle measures in acute triangles
- Develop and use the cosine law to determine side lengths and angle measures in acute triangles
- Solve problems that can be modelled using acute triangles

? Commercial ships transport goods from city to city around the Great Lakes.

Why can you not use a primary trigonometric ratio to directly calculate the distance by ship from St. Catharines to Toronto?

8 Getting Started

WORDS YOU NEED to Know

i) sine
ii) an acute triangle
iii) a right triangle
iv) cosine
v) hypotenuse
vi) Pythagorean theorem

1. Complete each sentence using one of the terms at the left.

a) A triangle in which each interior angle is less than 90° is called ________.

b) A triangle that contains a 90° angle is called ________.

c) The longest side in a right triangle is called the ________.

d) In a right triangle, the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ is called ________.

e) The ______________ describes the relationship between the three sides in a right triangle.

f) In a right triangle, the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ is called ________.

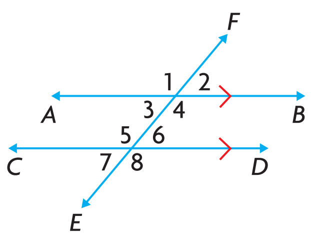
SKILLS AND CONCEPTS You Need

Study *Aid*

- For more help and practice, see Appendix A-15.

Angle Relationships

The properties of triangles, including the relationships between angles formed by **transversals** and parallel lines, can be used to determine unknown angle measures.



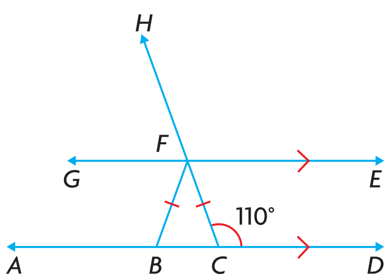
Alternate angles are equal:
$\angle 3 = \angle 6$ and $\angle 4 = \angle 5$

Corresponding angles are equal:
$\angle 1 = \angle 5$, $\angle 2 = \angle 6$,
$\angle 3 = \angle 7$, and $\angle 4 = \angle 8$

Co-interior angles are supplementary:
$\angle 4 + \angle 6 = 180°$
$\angle 3 + \angle 5 = 180°$

EXAMPLE

Determine all the unknown angle measures. Explain your reasons.

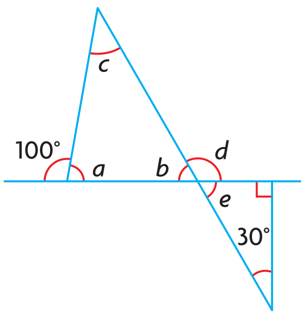


Solution

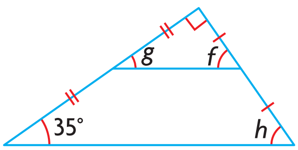
Angle Measure	Reason
$\angle FCB = 180° - \angle FCD$ $= 180° - 110°$ $= 70°$	$\angle FCB$ and $\angle FCD$ are supplementary angles.
$\angle FBC = \angle FCB$ $= 70°$	$FB = FC$, so $\triangle FBC$ is isosceles.
$\angle BFC = 180° - \angle FCB - \angle FBC$ $= 180° - 70° - 70°$ $= 40°$	Sum of the interior angles of a triangle is 180°.
$\angle FBA = 180° - \angle FBC$ $= 180° - 70°$ $= 110°$	$\angle FBA$ and $\angle FBC$ are supplementary angles.
$\angle EFC = \angle FCB$ $= 70°$	$GE \parallel AD$, so alternate angles are equal.
$\angle GFB = \angle FBC$ $= 70°$	$GE \parallel AD$, so alternate angles are equal.
$\angle HFE = \angle FCD$ $= 110°$	$GE \parallel AD$, so corresponding angles are equal.
$\angle HFG = 180° - \angle HFE$ $= 180° - 110°$ $= 70°$	$\angle HFG$ and $\angle HFE$ are supplementary angles.

2. Determine the measures of the indicated angles in each diagram.

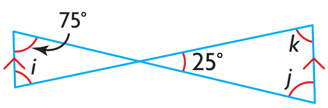
a)



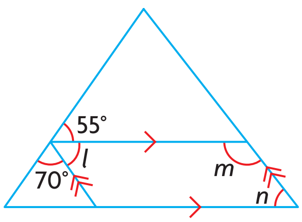
c)



b)



d)



Study Aid

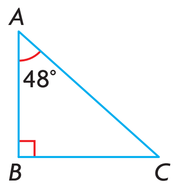
- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
3	A-15
6	A-14

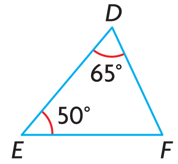
PRACTICE

3. Which is the longest side and which is the shortest side in each triangle?

a)

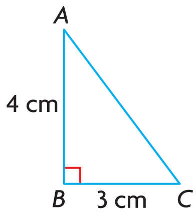


b)

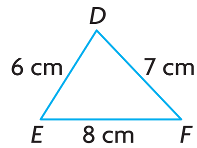


4. Which is the greatest angle and which is the least angle in each triangle?

a)



b)



5. Determine the value of each trigonometric ratio to four decimal places.

a) $\sin 55°$ **c)** $\cos 82°$

b) $\cos 24°$ **d)** $\sin 37°$

6. Solve.

a) $\frac{5}{3} = \frac{x}{12}$ **c)** $\sin 30° = \frac{x}{12}$

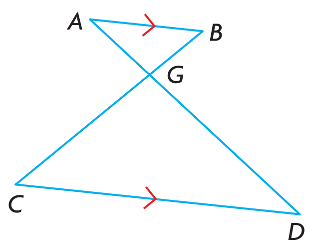
b) $\frac{36}{x} = \frac{9}{2}$ **d)** $\cos 60° = \frac{25}{x}$

7. Determine the measure of $\angle A$ to the nearest degree.

a) $\sin A = 0.5$ **c)** $\sin A = \frac{3}{4}$

b) $\cos A = 0.5$ **d)** $\cos A = \frac{5}{8}$

8. A 3 m board is leaning against a vertical wall. If the base of the board is placed 1 m from the wall, determine the measure of the angle that the board makes with the floor.



9. a) Is $\triangle ABG \sim \triangle DCG$ in the diagram at the left? Explain how you know.

b) Write the ratios that are equivalent to $\frac{AB}{DC}$.

10. Belinda claims that the value of any primary trigonometric ratio in a right triangle will always be less than 1. Do you agree or disagree? Justify your decision.

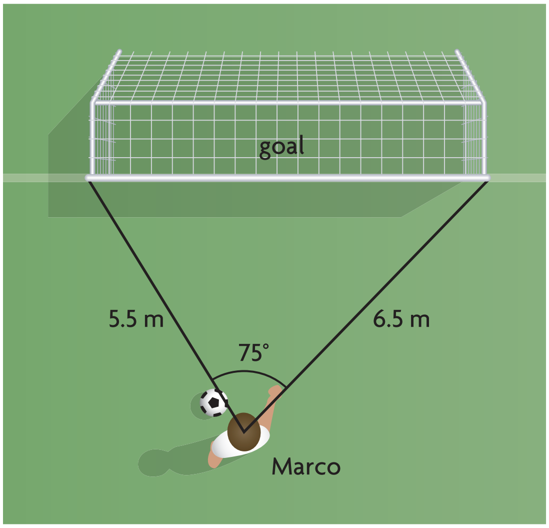
APPLYING What You Know

Soccer Trigonometry

YOU WILL NEED
- ruler
- protractor

Marco is about to take a shot in front of a soccer net. He estimates that his current position

- is 5.5 m from the left goalpost and 6.5 m from the right goalpost
- forms a 75° angle with the two goalposts



? How can you use these measurements to calculate the width of the soccer net?

A. Does Marco's position form a right triangle with the goalposts?

B. Can a primary trigonometric ratio be used to calculate the width of the net directly? Explain.

C. Copy the triangle in the diagram above. Add a line so that you can calculate the height of the triangle using a primary trigonometric ratio.

D. Calculate the height of the triangle.

E. Create and describe a plan that will allow you to calculate the width of the soccer net using the two right triangles you created.

F. Carry out your plan to calculate the width of the soccer net.

8.1 Exploring the Sine Law

YOU WILL NEED

- dynamic geometry software, or ruler and protractor

GOAL

Explore the relationship between each side in an acute triangle and the sine of its opposite angle.

EXPLORE the Math

The primary trigonometric ratios—sine, cosine, and tangent—are defined for right triangles.

? What is the relationship between a side and the sine of the angle that is opposite this side in an acute triangle?

Tech Support

For help using dynamic geometry software to construct a triangle, measure its angles and sides, and calculate, see Appendix B-25, B-26, B-29, and B-28.

A. Construct an **acute triangle**, $\triangle ABC$, and measure all its angles and sides to one decimal place. Record the measurements in a table like the one below.

Angle	Side	Sine	$\frac{\text{length of opposite side}}{\text{sin (angle)}}$
$\angle A =$	$a =$	$\sin A =$	$\frac{a}{\sin A} =$
$\angle B =$	$b =$	$\sin B =$	$\frac{b}{\sin B} =$
$\angle C =$	$c =$	$\sin C =$	$\frac{c}{\sin C} =$

B. Determine the sine of each angle. Calculate the ratio $\frac{\text{length of opposite side}}{\text{sin (angle)}}$ for each angle in the triangle. Record these values in your table.

C. What do you notice about the value of the ratio for each angle?

D. Create a different acute triangle, and repeat part B. What do you notice?

E. Create several more acute triangles, and repeat parts B and C.

F. Create a right triangle, and repeat part B. What do you notice?

G. Investigate whether replacing the sine ratio with the cosine or tangent ratio gives the same results.

H. Explain what you have discovered
i) in words **ii)** with a mathematical relationship

Reflecting

I. Suggest an appropriate name for the relationship you have discovered.

J. Does this relationship guarantee that if you know the measurements of an angle and the opposite side in a triangle, as well as the measurement of one other side or angle, you can calculate the measurements of the other angles and sides? Explain.

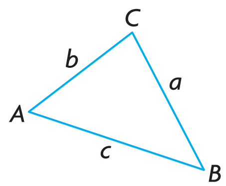
In Summary

Key Idea

- The ratio $\frac{\text{length of opposite side}}{\sin\text{(angle)}}$ is the same for all three angle–side pairs in an acute triangle.

Need to Know

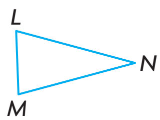
- In an acute triangle, $\triangle ABC$,
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
- This relationship is also true for right triangles.



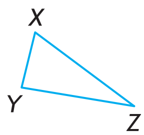
FURTHER Your Understanding

1. For each acute triangle,
i) copy the triangle and label the sides using lower-case letters
ii) write the ratios that are equivalent

a)



b)



2. Solve for the unknown side length or angle measure. Round your answer to one decimal place.

a) $\frac{w}{\sin 50°} = \frac{8.0}{\sin 60°}$

b) $\frac{k}{\sin 43°} = \frac{9.5}{\sin 85°}$

c) $\frac{6.0}{\sin M} = \frac{10.0}{\sin 72°}$

d) $\frac{12.5}{\sin Y} = \frac{14.0}{\sin 88°}$

3. Matt claims that if a and b are adjacent sides in an acute triangle, then $a \sin B = b \sin A$. Do you agree or disagree? Justify your decision.

4. If you want to calculate an unknown side length or angle measure in an acute triangle, what is the minimum information that you must have?

8.2 Applying the Sine Law

YOU WILL NEED

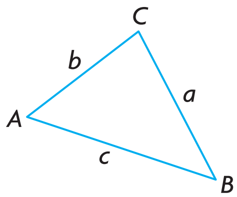
- ruler

GOAL

Use the sine law to calculate unknown side lengths and angle measures in acute triangles.

sine law

in any acute triangle,
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



LEARN ABOUT the Math

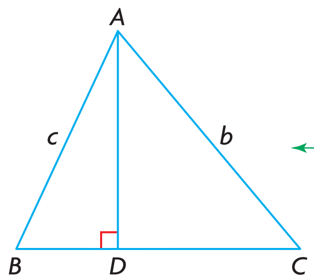
In Lesson 8.1, you discovered the **sine law** for acute triangles. Can you be sure that the sine law is true for every acute triangle?

? How can you show that the ratio $\frac{\text{length of opposite side}}{\sin(\text{angle})}$ is the same for all three angle-side pairs in any acute triangle?

EXAMPLE 1 Proving the sine law for acute triangles

Show that the sine law is true for all acute triangles.

Ben's Solution



I drew an acute triangle. Since the primary trigonometric ratios are used for right triangles only, I drew a perpendicular height, *AD*, from *A* to *BC* to form two right triangles, △*ABD* and △*ACD*.

In △ABD,
$$\sin B = \frac{AD}{c}$$

In △ACD,
$$\sin C = \frac{AD}{b}$$

Since the sine law involves the sine of the angles in △*ABC*, I wrote equations for the sines of ∠*B* and ∠*C* in the two right triangles using the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$.

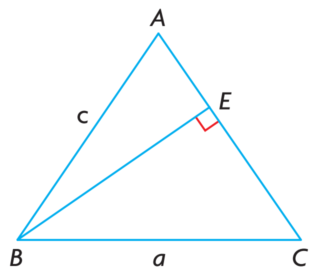
$$c \sin B = AD \qquad b \sin C = AD$$

I knew that *AD* was the opposite side in both right triangles, so I used the sine ratio to describe *AD* in △*ABD* and △*ACD*. I thought this would allow me to relate the two triangles.

$$c \sin B = b \sin C$$
$$\frac{c \sin B}{\sin C} = b$$
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

Since the expressions $c \sin B$ and $b \sin C$ both describe AD, I set them equal.

I wanted to write each side of the equation as a ratio with information about only one triangle. I divided both sides of the equation by $\sin C$ and then by $\sin B$.



In $\triangle ABE$,

$$\sin A = \frac{BE}{c}$$
$$c \sin A = BE$$

In $\triangle CBE$,

$$\sin C = \frac{BE}{a}$$
$$a \sin C = BE$$

I wanted to show that the ratios $\frac{c}{\sin C}$ and $\frac{b}{\sin B}$ were equal to $\frac{a}{\sin A}$.

To do this, I needed to divide $\triangle ABC$ differently so that I could use $\angle A$. I reasoned that if I drew a height BE from B to AC, I could follow the same steps I used for height AD.

$$c \sin A = a \sin C$$
$$c = \frac{a \sin C}{\sin A}$$
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

Since the expressions $c \sin A$ and $a \sin C$ both describe BE, I set them equal. Then I divided both sides of the equation by $\sin A$ and then by $\sin C$.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Since $\frac{b}{\sin B}$ and $\frac{a}{\sin A}$ are equal to $\frac{c}{\sin C}$, all three ratios must be equal.

Reflecting

A. Why did Ben need to draw line segment AD perpendicular to side BC?

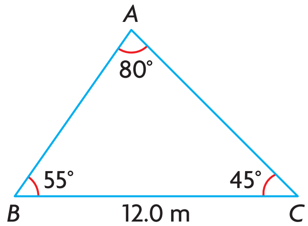
B. If Ben drew a perpendicular line segment from vertex C to side AB, which pair of ratios in the sine law do you think he could show are equal?

C. Why does it make sense that the sine law can also be written in the form $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$?

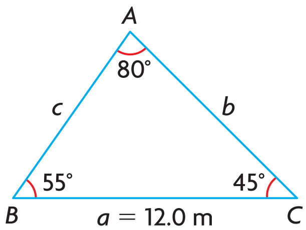
APPLY the Math

EXAMPLE 2 | **Selecting a sine law strategy** to calculate the length of a side

Determine the length of AC.



Elizabeth's Solution



I named the sides of the triangle using lower-case letters that correspond to the opposite angles. Then I identified the measures I didn't know.

Since the triangle does not contain a right angle, I couldn't use the primary trigonometric ratios.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12.0}{\sin 80°} = \frac{b}{\sin 55°}$$
$$\sin 55°\left(\frac{12.0}{\sin 80°}\right) = \sin 55°\left(\frac{b}{\sin 55°}\right)$$
$$\sin 55°\left(\frac{12.0}{\sin 80°}\right) = b$$
$$0.8192\left(\frac{12.0}{0.9848}\right) \doteq b$$
$$9.98 \doteq b$$

I realized that I could use the sine law if I knew an opposite side-angle pair, plus one more side or angle in the triangle. I knew a and $\angle A$ and I wanted to know b, so I related a, b, $\sin A$, and $\sin B$. I could have used $\frac{\sin A}{a} = \frac{\sin B}{b}$, but I decided to use $\frac{a}{\sin A} = \frac{b}{\sin B}$ since I had to solve for b. This meant that b was the numerator and I could multiply both sides by sin 55° to solve for b.

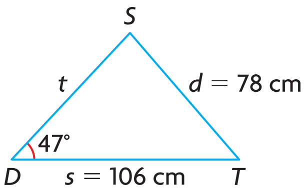
The length of AC is about 10.0 m.

It made sense that b is shorter than a, since $\angle B$ is less than $\angle A$.

EXAMPLE 3 | **Selecting a sine law strategy** to calculate the measure of an angle

In $\triangle DST$, $\angle D = 47°$, $d = 78$ cm, and $s = 106$ cm. Determine the measure of $\angle S$.

Phil's Solution



I drew a diagram. I knew that $\angle S$ was greater than $\angle D$ since $s > d$. I couldn't assume that the triangle contained a right angle, so I couldn't use the primary trigonometric ratios.

$$\frac{\sin S}{s} = \frac{\sin D}{d}$$

$$\frac{\sin S}{106} = \frac{\sin 47°}{78}$$

$$106\left(\frac{\sin S}{106}\right) = 106\left(\frac{\sin 47°}{78}\right)$$

$$\sin S = 106\left(\frac{\sin 47°}{78}\right)$$

I knew s, d, and $\angle D$, and I wanted to determine the measure of $\angle S$. So, I used the sine law that included these four quantities. I used the proportion with sin S and sin D in the numerators to make the equation easier to solve for sin S.

$$\sin S \doteq 0.9939$$

$$\angle S = \sin^{-1}(0.9939)$$

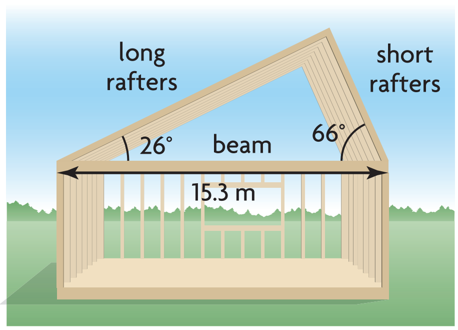
$$\angle S \doteq 83.7°$$

To determine $\angle S$, I calculated the inverse of the sine ratio.

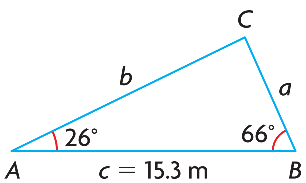
$\angle S$ is about 84°.

EXAMPLE 4 Solving a problem using the sine law

The roof of a new house must be built to exact specifications so that solar panels can be installed. The long rafters at the front of the house must be inclined at an angle of 26° to the horizontal beam. The short rafters at the back of the house must be inclined at an angle of 66°. The house is 15.3 m wide. Determine the length of the long rafters.



Taylor's Solution



I drew an acute triangle to model the situation. The long rafters were opposite $\angle B$, the 66° angle, so I labelled them side b. I knew that my diagram was correct, and the long rafters were side b, since $66° > 26°$.

$$\angle C = 180° - 26° - 66°$$

$$= 88°$$

Since I knew length c, I had to use ratios that involved $\angle B$ and $\angle C$. So, I had to determine the measure of $\angle C$. I knew that the angles in a triangle add up to 180°.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 66°} = \frac{15.3}{\sin 88°}$$

Since I needed to determine side *b*, I used the sine law to write a proportion with the sides in the numerator.

$$\sin 66°\left(\frac{b}{\sin 66°}\right) = \sin 66°\left(\frac{15.3}{\sin 88°}\right)$$

I solved for *b*.

$$b = \sin 66°\left(\frac{15.3}{\sin 88°}\right)$$
$$b \doteq 13.99$$

Since $\angle B$ is less than $\angle C$, it makes sense that *b* is shorter than *c*.

The long rafters are about 14.0 m long.

In Summary

Key Idea

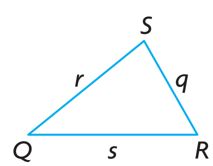
- The sine law can be used to determine unknown side lengths or angle measures in some acute triangles.

Need to Know

- To use the sine law to determine a side length or angle measure, follow these steps:
  - Determine the ratio of the sine of a known angle measure and a known side length.
  - Create an equivalent ratio using the unknown side length and the measure of its opposite angle, or the sine of the unknown angle measure and the length of its opposite side.
  - Equate the ratios you created, and solve.
- You can use the sine law to solve a problem modelled by an acute triangle if you know the measurements of
  - two sides and the angle that is opposite one of these sides
  - two angles and any side
- An acute triangle can be divided into right triangles. The proof of the sine law involves writing proportions that compare corresponding sides in these right triangles.

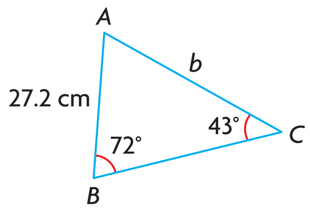
CHECK Your Understanding

1. Write three equivalent ratios using the sides and angles in the triangle at the right.

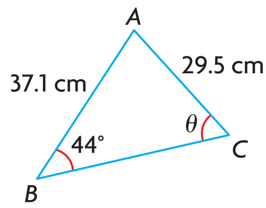


2. Determine the indicated measures to one decimal place.

a)



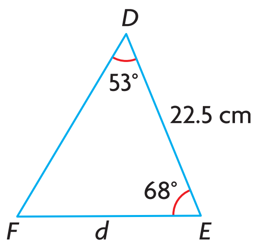
b)



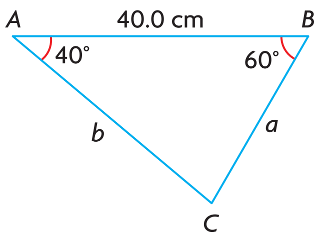
PRACTISING

3. Determine the indicated side lengths and angle measures.

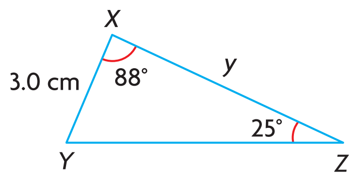
a)



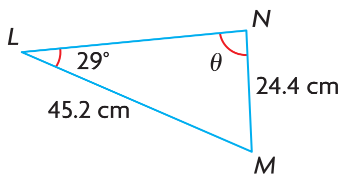
b)



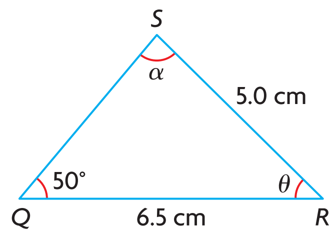
c)



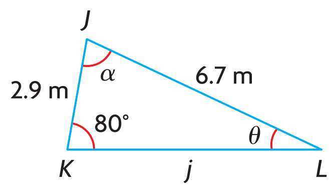
d)



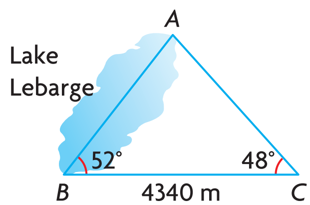
e)



f)



4. **A** Scott is a naturalist. He is studying the effects of acid rain on fish populations in different lakes. As part of his research, he needs to know the length of Lake Lebarge. Scott makes the measurements shown. How long is Lake Lebarge?



5. Draw a labelled diagram for each triangle. Then calculate the required side length or angle measure.

a) In $\triangle SUN$, $n = 58$ cm, $\angle N = 38°$, and $\angle U = 72°$. Determine the length of side u.

b) In $\triangle PQR$, $\angle R = 73°$, $\angle Q = 32°$, and $r = 23$ cm. Determine the length of side q.

c) In $\triangle TAM$, $t = 8$ cm, $m = 6$ cm, and $\angle T = 65°$. Determine the measure of $\angle M$.

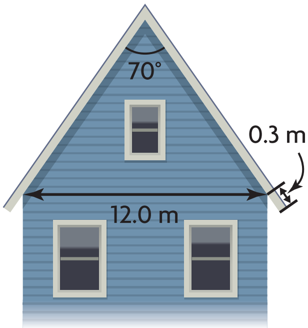
d) In $\triangle WXY$, $w = 12.0$ cm, $y = 10.5$ cm, and $\angle W = 60°$. Determine the measure of $\angle Y$.

6. **K** In $\triangle CAT$, $\angle C = 32°$, $\angle T = 81°$, and $c = 24.1$ m. Solve the triangle.

Environment *Connection*

Acidic lakes cannot support the variety of life in healthy lakes. Clams and crayfish are the first to disappear, followed by other species of fish.

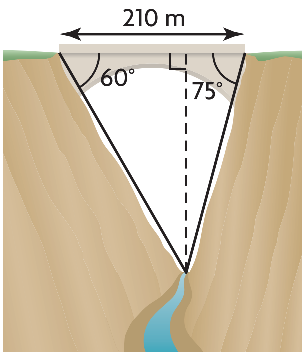
7. The short sides of a parallelogram are both 12.0 cm. The acute angles of the parallelogram are 65°, and the short diagonal is 15.0 cm. Determine the length of the long sides of the parallelogram. Round your answer to the nearest tenth of a centimetre.



8. An architect designed a house that is 12.0 m wide. The rafters that hold up the roof are equal in length and meet at an angle of 70°, as shown at the left. The rafters extend 0.3 m beyond the supporting wall. How long are the rafters?

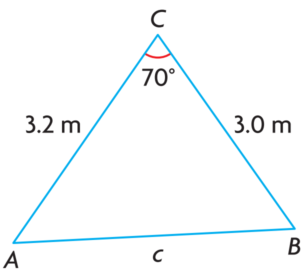
9. A telephone pole is supported by two wires on opposite sides. **T** At the top of the pole, the wires form an angle of 60°. On the ground, the ends of the wires are 15.0 m apart. One wire makes a 45° angle with the ground. How long are the wires, and how tall is the pole?

10. In $\triangle PQR$, $\angle Q = 90°$, $r = 6$, and $p = 8$. Explain two different ways to calculate the measure of $\angle P$.



11. A bridge across a gorge is 210 m long, as shown in the diagram at the left. The walls of the gorge make angles of 60° and 75° with the bridge. Determine the depth of the gorge to the nearest metre.

12. Use the sine law to help you describe each situation.
C a) Three pieces of information allow you to solve for all the unknown side lengths and angle measures in a triangle.
b) Three pieces of information do not allow you to solve a triangle.



13. Jim says that the sine law cannot be used to determine the length of side c in $\triangle ABC$ at the left. Do you agree or disagree? Explain.

14. Suppose that you know the length of side p in $\triangle PQR$, as well as the measures of $\angle P$ and $\angle Q$. What other sides and angles could you calculate? Explain how you would determine these measurements.

Extending

15. In $\triangle ABC$, $\angle A = 58°$, $\angle C = 74°$, and $b = 6$. Calculate the area of $\triangle ABC$ to one decimal place.

16. An isosceles triangle has two sides that are 10 cm long and two angles that measure 50°. A line segment bisects one of the 50° angles and ends at the opposite side. Determine the length of the line segment.

17. Use the sine law to write a ratio that is equivalent to each expression for $\triangle ABC$.

a) $\dfrac{a}{\sin A}$ b) $\dfrac{\sin A}{\sin B}$ c) $\dfrac{a}{c}$ d) $\dfrac{a \sin C}{c \sin A}$

8 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: What is the sine law, and what is it used for?

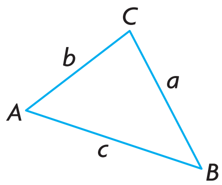
A: The sine law describes the relationship between sides and their opposite angles in a triangle. According to the sine law, the ratio $\frac{\text{length of opposite side}}{\sin(\text{angle})}$ is the same for all three angle-side pairs in a triangle.

In $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The sine law can be used to determine unknown side lengths and angle measures in acute and right triangles.

Study Aid

- See Lessons 8.1 and 8.2.
- Try Mid-Chapter Review Questions 1 to 3.

Q: When can you use the sine law?

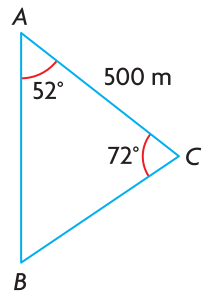
A: You can use the sine law if you know any three of these four measurements: two side lengths and the measures of their opposite angles. The sine law will allow you to calculate the fourth side length or angle measure. If you know any two angle measures in a triangle, you can calculate the third angle measure.

Study Aid

- See Lesson 8.2, Examples 2 to 4.
- Try Mid-Chapter Review Questions 4 to 9.

EXAMPLE

Can the sine law be used to determine the length of AB in the triangle at the right? Explain.



Solution

Yes.

Side AB (or c) is opposite $\angle C$, which equals 72°.

Side AC (or b) equals 500 m and is opposite $\angle B$.

$$\angle B = 180° - 52° - 72°$$
$$= 56°$$

Solving for c in this proportion gives the length of AB.

$$\frac{c}{\sin 72°} = \frac{500}{\sin 56°}$$
$$c = \sin 72°\left(\frac{500}{\sin 56°}\right)$$
$$c \doteq 573.6$$

The length of AB is about 574 m.

PRACTICE Questions

Lesson 8.1

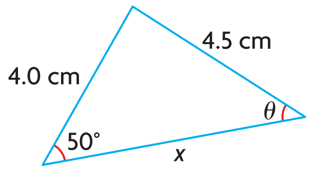
1. What relationship(s) does the sine law describe in the acute triangle XYZ?

2. Why are you more likely to use the sine law for acute triangles than for right triangles?

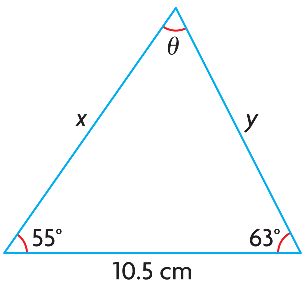
3. $\triangle DEF$ is an acute triangle. Nazir claims that $\frac{d}{\sin F} = \frac{f}{\sin D}$. Do you agree or disagree? Explain.

Lesson 8.2

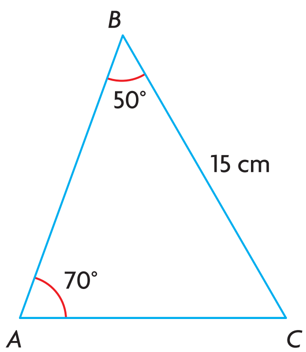
4. a) Determine the measure of θ and the length of side x.



b) Determine the measure of θ and the lengths of sides x and y.

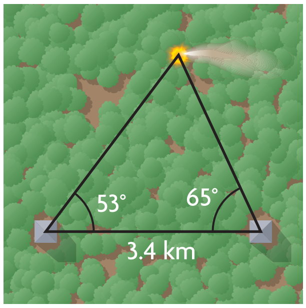


5. In $\triangle ABC$, $\angle A = 70°$, $\angle B = 50°$, and $a = 15$ cm. Solve $\triangle ABC$.



6. In $\triangle XYZ$, the values of x and z are known. What additional information do you need to know if you want to use the sine law to solve the triangle?

7. Two fire towers in a park are 3.4 km apart. When the park rangers on duty spot a fire, they can locate the fire by measuring the angle between the fire and the other tower. A fire is located 53° from one tower and 65° from the other tower.



a) Which tower is closer to the fire?

b) Determine the distance from the closest tower to the fire.

8. As Chloe and Ivan canoe across a lake, they notice a campsite ahead at an angle of 22° to the left of their direction of paddling. After continuing to paddle in the same direction for 800 m, the campsite is behind them at an angle of 110° to their direction of paddling. How far away is the campsite at the second sighting?

9. Calculate the perimeter of an isosceles triangle with

a) a base of 25 cm and only one angle of 50°

b) a base of 30 cm and equal angles of 55°

8.3 Exploring the Cosine Law

GOAL

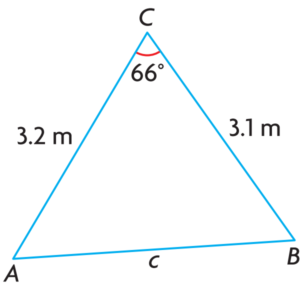
Explore the relationship between side lengths and angle measures in a triangle using the cosines of angles.

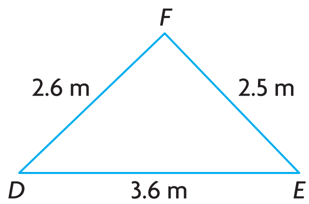
YOU WILL NEED

- dynamic geometry software

EXPLORE the Math

Sometimes, you cannot use the sine law to determine an unknown side length or angle measure in an acute triangle. This occurs when you do not have enough information to write a ratio comparing a side length and its opposite angle. For example, consider these two triangles:



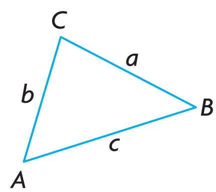


Tech | *Support*

For help using dynamic geometry software to construct a triangle, measure its angles and sides, and calculate, see Appendix B-25, B-26, B-29, and B-28.

? How can the Pythagorean theorem be extended to relate the sides and angles in these two triangles?

A. Use dynamic geometry software to construct any acute triangle. Label the vertices A, B, and C and the sides a, b, and c, as in the triangle at the right.



B. Measure all three interior angles and all three sides.

C. Drag a vertex until $\angle C = 90°$. State the Pythagorean relationship for this triangle.

D. Calculate.

i) $a^2 + b^2$ **iii)** $a^2 + b^2 - c^2$

ii) c^2 **iv)** $2\,ab \cos C$

Record your results in a table like the one shown.

Triangle	a	b	c	$\angle C$	c^2	$a^2 + b^2$	$a^2 + b^2 - c^2$	$2\,ab \cos C$
1				90°				
2								
3								
4								
5								

Communication | *Tip*

2 ab cos C is a product in which four terms are multiplied: (2)(a)(b)(cos C).

E. Drag vertex C away from side AB to create a new acute triangle. Repeat part D for this triangle.

F. What do you notice about your results for $a^2 + b^2 - c^2$ and $2\,ab \cos C$?

G. Drag vertex C to at least five other positions. Repeat part D for each new triangle you create.

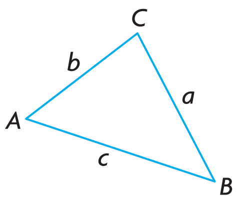
H. Based on your observations, what can you conclude about the relationship among a^2, b^2, and c^2 in an acute triangle?

Reflecting

I. When $\angle C = 90°$, what happens to the value of $2\,ab \cos C$ in the **cosine law**? Why does this happen?

cosine law

in any acute triangle, $c^2 = a^2 + b^2 - 2\,ab \cos C$



J. How does the measure of $\angle C$ affect the value of $2\,ab \cos C$?

K. Explain how the cosine law could be used to relate
- **i)** the value of a^2 to the value of $b^2 + c^2$
- **ii)** the value of b^2 to the value of $a^2 + c^2$

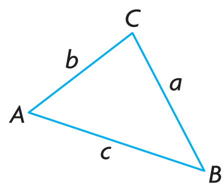
In Summary

Key Idea

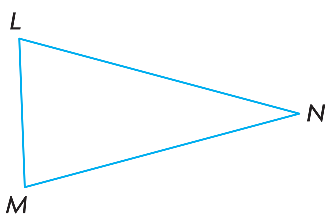
- The cosine law is an extension of the Pythagorean theorem to triangles with no right angle.

Need to Know

- The cosine law states that for any $\triangle ABC$,
  $a^2 = b^2 + c^2 - 2\,bc \cos A$
  $b^2 = a^2 + c^2 - 2\,ac \cos B$
  $c^2 = a^2 + b^2 - 2\,ab \cos C$



FURTHER Your Understanding



1. a) Sketch $\triangle LMN$ at the left, and label each side using lower-case letters.

b) Relate each side length in $\triangle LMN$ to the cosine of its opposite angle and the lengths of the other two sides.

2. Sketch each triangle, and then solve for the unknown side length or angle measure.

a) $w^2 = 15^2 + 16^2 - 2(15)(16)\cos 75°$

b) $k^2 = 32^2 + 35^2 - 2(32)(35)\cos 50°$

c) $48^2 = 46^2 + 45^2 - 2(46)(45)\cos Y$

d) $13^2 = 17^2 + 15^2 - 2(17)(15)\cos G$

3. Martina used algebra to write the cosine law as follows:

$$r^2 = p^2 + q^2 - 2\,pq \cos R$$
$$r^2 - p^2 - q^2 = -2\,pq \cos R$$
$$\frac{r^2 - p^2 - q^2}{-2\,pq} = \frac{-2\,pq \cos R}{-2\,pq}$$
$$\frac{p^2 + q^2 - r^2}{2\,pq} = \cos R$$

a) Explain the advantage of writing the cosine law this way.
b) Write the cosine law for cos P in terms of sides p, q, and r.
c) Write the cosine law for cos Q in terms of sides p, q, and r.

4. Identify what you need to know about a triangle if you want to use the cosine law to calculate
a) an unknown side length
b) an unknown angle measure

5. Express the cosine law in words to describe the relationship between the three sides in an acute triangle and the cosine of one angle.

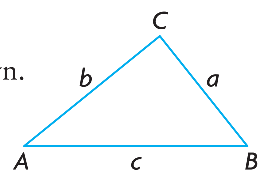
Curious | Math

YOU WILL NEED
- dynamic geometry software

The Law of Tangents

You have seen that the sine law and cosine law relate the sines and cosines of angles in acute and right triangles to the measures of their sides. Does a similar relationship exist for the tangents of the angles in a triangle?

1. Using dynamic geometry software, construct an acute triangle. Label the angles and sides as shown.



2. Measure all three sides and all three angles.

3. Select the lengths of a and b. Determine the value of the ratio $\dfrac{a - b}{a + b}$.

4. Select the measures of $\angle A$ and $\angle B$. Determine the value of the ratio $\dfrac{\tan\left(\frac{1}{2}(A - B)\right)}{\tan\left(\frac{1}{2}(A + B)\right)}$. What do you notice?

5. Make a conjecture about what the tangent law for triangles might be. Test your conjecture by dragging one of the vertices to a new position. Repeat this two more times, using a different vertex each time.

6. Write the law of tangents in terms of
a) $\angle B$, $\angle C$, and sides b and c
b) $\angle A$, $\angle C$, and sides a and c

8.4 Applying the Cosine Law

YOU WILL NEED

- ruler

GOAL

Use the cosine law to calculate unknown measures of sides and angles in acute triangles.

LEARN ABOUT the Math

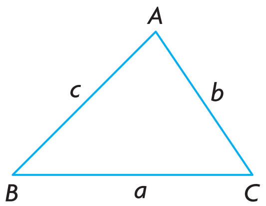
In Lesson 8.3, you discovered the cosine law for acute triangles. Can you be sure that the cosine law is true for every acute triangle?

? How can you show that the cosine law is true for all acute triangles?

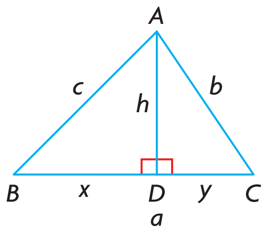
EXAMPLE 1 **Proving** the cosine law for acute triangles

Show that the cosine law is true for all acute triangles.

Heather's Solution



I started by drawing an acute triangle ABC.



The cosine law is just an extension of the Pythagorean theorem. I thought that I might be able to relate the angles and sides in $\triangle ABC$ if I could create right triangles. I drew a line segment from A to D so that it was perpendicular to BC. I labelled this line segment h. I used x for BD and y for DC.

$$c^2 = h^2 + x^2, \text{ so } h^2 = c^2 - x^2$$
$$b^2 = h^2 + y^2, \text{ so } h^2 = b^2 - y^2$$

I wrote the Pythagorean theorem for each triangle to determine two different expressions for h^2.

$$c^2 - x^2 = b^2 - y^2$$

Then I set the two expressions equal.

$$x = a - y, \text{ so}$$
$$c^2 - (a - y)^2 = b^2 - y^2$$
$$c^2 = (a - y)^2 + b^2 - y^2$$
$$c^2 = a^2 - 2ay + y^2 + b^2 - y^2$$
$$c^2 = a^2 + b^2 - 2ay$$

I didn't want to include both x and y in the same equation. I only wanted one of them. So, I substituted $x = a - y$ for x. I simplified the equation by expanding and collecting like terms.

$$\cos C = \frac{y}{b}, \text{ so}$$
$$b \cos C = y$$

The cosine law for this triangle had to include side lengths a, b, and c, as well as one angle. My equation $c^2 = a^2 + b^2 - 2ay$ included the three side lengths, but it also included y, which I didn't know. My equation did not involve any angles.

I had to write y in terms of one of the angles in the triangle. Since y is adjacent to $\angle C$ in $\triangle ADC$, I decided to write y in terms of the cosine of $\angle C$.

$$c^2 = a^2 + b^2 - 2ay$$
$$c^2 = a^2 + b^2 - 2\,ab \cos C$$

I substituted the expression $b \cos C$ for y into my equation.

Reflecting

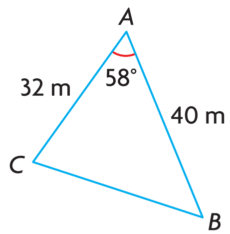
A. Why did it make sense for Heather to divide the acute triangle into two right triangles?

B. Suppose that Heather had substituted $a - x$ for y instead of $a - y$ for x. Would her result have been the same? How do you know?

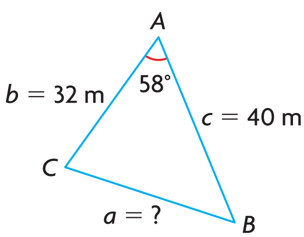
APPLY the Math

EXAMPLE 2 Selecting a cosine law strategy to calculate the length of a side

Determine the length of CB.



Justin's Solution



I copied the triangle and named the sides using lower-case letters. Then I identified the measure that I had to determine. Since the triangle did not contain a right angle, I couldn't use primary trigonometric ratios. I couldn't use the sine law either, because I didn't know a side length and the measure of its opposite angle.

$$a^2 = b^2 + c^2 - 2\,bc \cos A$$
$$a^2 = 32^2 + 40^2 - 2(32)(40)\cos 58°$$

I knew two sides (b and c) and the angle between these sides ($\angle A$). I had to determine side a, which is opposite $\angle A$. The cosine law relates these four measurements, so I substituted the values I knew into the cosine law.

$$a^2 = 1024 + 1600 - 2560 \cos 58°$$
$$a^2 = 2624 - 2560 \cos 58°$$
$$a^2 = 1267.41$$
$$a = \sqrt{1267.41}$$
$$a \doteq 35.6$$

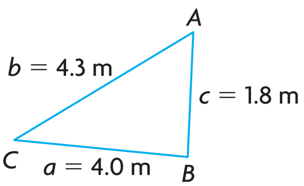
I evaluated the right side. Then I calculated the square root.

CB is about 36 m.

EXAMPLE 3 **Selecting a cosine law strategy** to calculate the measure of an angle

The posts of a hockey net are 1.8 m apart. A player tries to score a goal by shooting the puck along the ice from a point that is 4.3 m from one goalpost and 4.0 m from the other goalpost. Determine the measure of the angle that the puck makes with both goalposts.

Darcy's Solution



I drew a diagram to represent the situation. I couldn't assume that this triangle contained a right angle, so I couldn't use primary trigonometric ratios. I didn't know the measure of an angle and a side length opposite the angle, so I couldn't use the sine law.

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$1.8^2 = 4.0^2 + 4.3^2 - 2(4.0)(4.3) \cos C$$

I had to determine the measure of $\angle C$. The cosine law relates the three sides of a triangle to an angle in the triangle. I substituted the measures of the sides in this triangle into the cosine law.

$$3.24 = 16.00 + 18.49 - 34.40 \cos C$$
$$3.24 - 16.00 - 18.49 = -34.40 \cos C$$
$$-31.25 = -34.40 \cos C$$
$$\frac{-31.25}{-34.40} = \cos C$$
$$0.9084 \doteq \cos C$$
$$\cos^{-1}(0.9084) = \angle C$$
$$24.7° \doteq \angle C$$

I simplified and then solved for $\cos C$.

I used the inverse cosine to calculate $\angle C$.

The puck makes an angle of about 25° with the goalposts.

In Summary

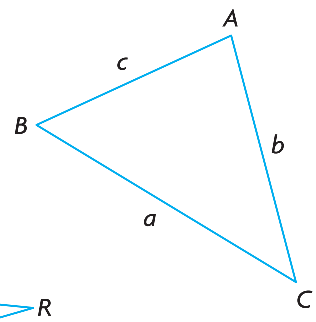
Key Idea

- The cosine law can be used to determine an unknown side length or angle measure in an acute triangle.

Need to Know

- You can use the cosine law to solve a problem that can be modelled by an acute triangle if you can determine the measurements of
  - two sides and the angle between them
  - all three sides
- An acute triangle can be divided into smaller right triangles by drawing a perpendicular line from a vertex to the opposite side. The proof of the cosine law involves applying the Pythagorean theorem and cosine ratio to these right triangles.

CHECK *Your Understanding*

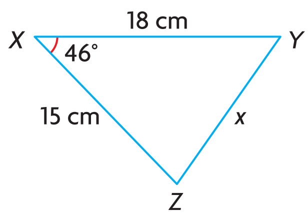


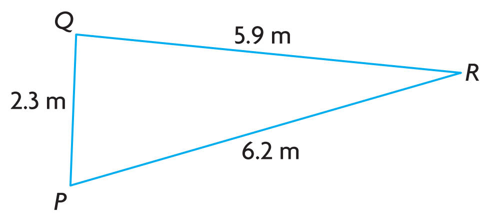
1. Suppose that you are given each set of data for $\triangle ABC$ at the right. Can you use the cosine law to determine c? Explain.

a) $a = 5$ cm, $\angle A = 52°$, $\angle C = 43°$

b) $a = 5$ cm, $b = 7$ cm, $\angle C = 43°$

2. a) Determine the length of side x. **b)** Determine the measure of $\angle P$.

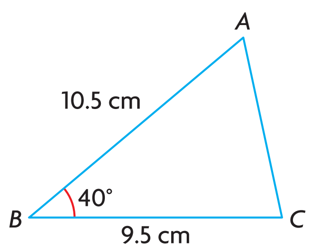




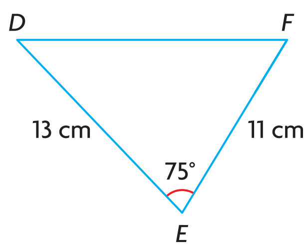
PRACTISING

3. Determine each unknown side length.

a)

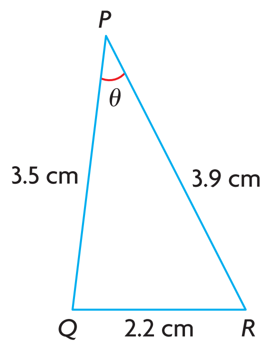


b)

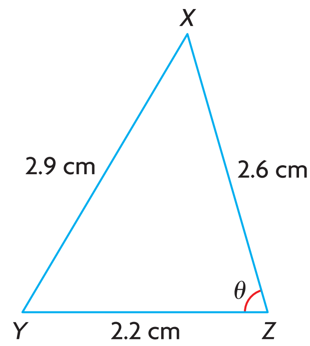


4. Determine the measure of each indicated angle to the nearest degree.

a)



b)



5. Solve each triangle.

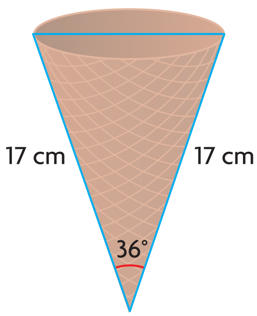
K **a)** In $\triangle DEF$, $d = 5.0$ cm, $e = 6.5$ cm, and $\angle F = 65°$.

b) In $\triangle PQR$, $p = 6.4$ m, $q = 9.0$ m, and $\angle R = 80°$.

c) In $\triangle LMN$, $l = 5.5$ cm, $m = 4.6$ cm, and $n = 3.3$ cm.

d) In $\triangle XYZ$, $x = 5.2$ mm, $y = 4.0$ mm, and $z = 4.5$ cm.

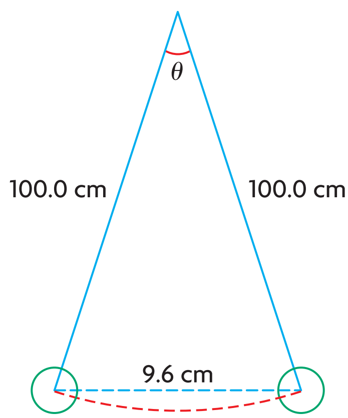
6. Determine the perimeter of $\triangle SRT$, if $\angle S = 60°$, $r = 15$ cm, and $t = 20$ cm.



7. An ice cream company is designing waffle cones to use for serving frozen yogurt. The cross-section of the design has a bottom angle of 36°. The sides of the cone are 17 cm long. Determine the diameter of the top of the cone.

8. **C** A parallelogram has sides that are 8 cm and 15 cm long. One of the angles in the parallelogram measures 70°. Explain how you could calculate the length of the shortest diagonal.

9. **A** The pendulum of a grandfather clock is 100.0 cm long. When the pendulum swings from one side to the other side, the horizontal distance it travels is 9.6 cm, as in the diagram at the right. Determine the angle through which the pendulum swings. Round your answer to the nearest tenth of a degree.

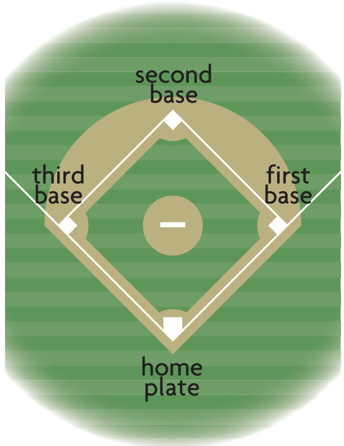


10. **a)** A clock has a minute hand that is 20 cm long and an hour hand that is 12 cm long. Calculate the distance between the tips of the hands at

i) 2:00 **ii)** 10:00

b) Discuss your results for part a).

11. The bases in a baseball diamond are 90 ft apart. A player picks up a ground ball 11 ft from third base, along the line from second base to third base. Determine the angle that is formed between first base, the player's present position, and home plate.



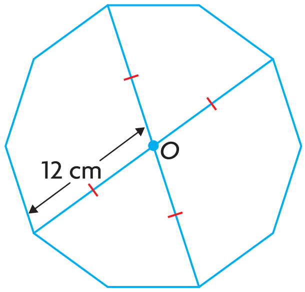
History *Connection*

The first baseball game recorded in Canada was played in Beachville, Ontario, on June 4, 1838.

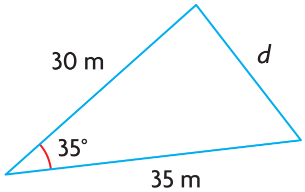
12. Sally makes stained glass windows. Each piece of glass is surrounded by lead edging. Sally claims that she can create an acute triangle in part of a window using pieces of lead that are 15 cm, 36 cm, and 60 cm. Is she correct? Justify your decision.

13. Two drivers leave home at the same time and travel on straight roads that diverge by 70°. One driver travels at an average speed of 83.0 km/h. The other driver travels at an average speed of 95.0 km/h. How far apart will the two drivers be after 45 min?

14. T The distance from the centre, O, of a regular decagon to each vertex is 12 cm. Calculate the area of the decagon.



15. Use the triangle at the right to create a problem that involves side lengths and interior angles. Then describe how to determine the length of side d.



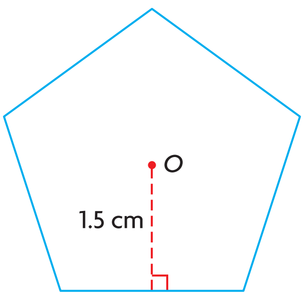
Extending

16. An airplane is flying from Montréal to Vancouver. The wind is blowing from the west at 60 km/h. The airplane flies at an airspeed of 750 km/h and must stay on a heading of 65° west of north.

a) What heading should the pilot take to compensate for the wind?

b) What is the speed of the airplane relative to the ground?

17. Calculate the perimeter and area of this regular pentagon. O is the centre of this pentagon.



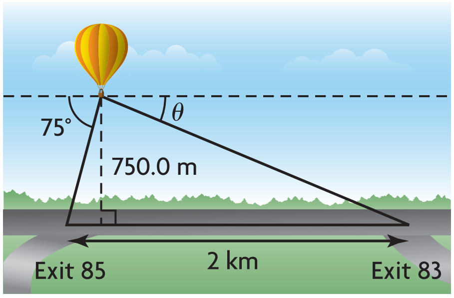
8.5 Solving Acute Triangle Problems

YOU WILL NEED

- ruler

GOAL

Solve problems using the primary trigonometric ratios and the sine and cosine laws.



LEARN ABOUT the Math

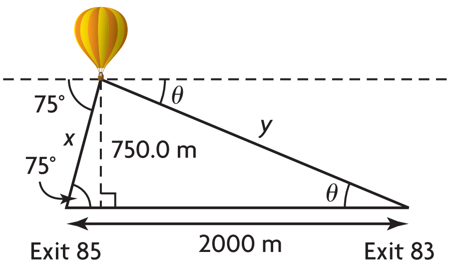
Reid's hot-air balloon is 750.0 m directly above a highway. When Reid is looking west, the angle of depression to Exit 85 is 75°. Exit 83 is located 2 km to the east of Exit 85.

? What is the angle of depression to Exit 83 when Reid is looking east?

EXAMPLE 1 **Solving a problem** using an acute triangle model

Determine the angle of depression, to the nearest degree, from the balloon to Exit 83.

Vlad's Solution



The ground and the horizontal are parallel, so the alternate angles are equal. The angle of elevation to the balloon at Exit 83 equals θ, the angle of depression I needed to determine. The distance between the exits is 2 km, or 2000 m.

I labelled the unknown sides of the large triangle x and y. If I could determine both x and y, then I could calculate θ using the sine ratio.

$$\sin 75° = \frac{750.0}{x}$$
$$x \sin 75° = 750.0$$
$$x = \frac{750.0}{\sin 75°}$$
$$x \doteq 776.46$$

In the right triangle that contains the 75° angle, x is the hypotenuse and the 750.0 m side is the opposite side. I used the sine ratio to write an equation. Then I solved for x.

$$y^2 = x^2 + 2000^2 - 2(x)(2000)\cos 75°$$
$$y^2 = 776.46^2 + 2000^2 - 2(776.46)(2000)\cos 75°$$
$$y^2 = 602\,890.1316 + 4\,000\,000 - 803\,850.543$$
$$y^2 = 3\,799\,039.589$$
$$y = \sqrt{3\,799\,039.589}$$
$$y \doteq 1949.11$$

I now knew the lengths of two sides in the large acute triangle and the angle between them. So, I was able to use the cosine law to determine y.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{750.0}{1949.11}$$
$$\sin\theta \doteq 0.3848$$

In the right triangle that contained θ, I knew the opposite side, 750.0 m, and the hypotenuse, y. With these values, I was able to determine the value of $\sin\theta$.

$$\theta = \sin^{-1}(0.3848)$$
$$\theta \doteq 22.6°$$

I used the inverse sine to calculate θ.

The angle of depression from the balloon to Exit 83 is about 23°.

I rounded my answer to the nearest degree.

Reflecting

A. Why do you think Vlad started by using the right triangle that contained x instead of the right triangle that contained y?

B. Vlad used the cosine law to determine y. Could he have used another strategy to determine y? Explain.

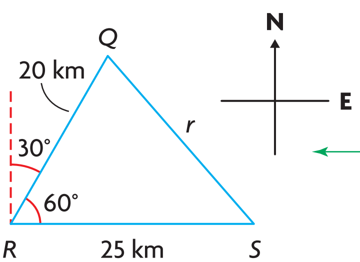
C. Could Vlad have calculated the value of θ using the sine law? Explain.

APPLY the Math

EXAMPLE 2 Solving a problem that involves directions

The captain of a boat leaves a marina and heads due west for 25 km. Then the captain adjusts the course of his boat and heads N30°E for 20 km. How far is the boat from the marina?

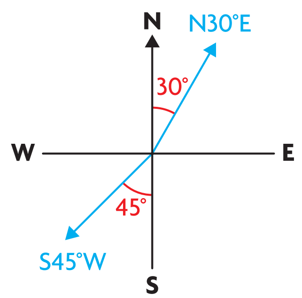
Audrey's Solution



I drew a diagram to represent this situation. The directions north and west are perpendicular to each other. Since N30°E means that the boat travels along a line 30° east of north, I was able to determine $\angle QRS$ by subtracting 30° from 90°.

Communication *Tip*

Directions are often stated in terms of north and south on a compass. For example, N30°E means travelling in a direction 30° east of north. S45°W means travelling in a direction 45° west of south.



$$r^2 = s^2 + q^2 - 2(s)(q)\cos R$$
$$r^2 = 20^2 + 25^2 - 2(20)(25)\cos 60°$$
$$r^2 = 525$$
$$r = \sqrt{525}$$
$$r \doteq 22.9$$

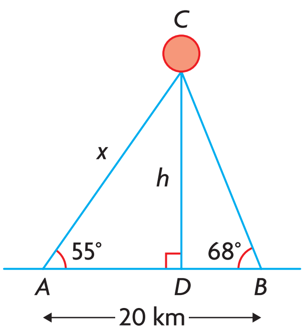
To determine r, I used the cosine law.

The boat is about 23 km from the marina.

EXAMPLE 3 Solving a problem using acute and right triangles

A weather balloon is directly between two tracking stations. The angles of elevation from the two tracking stations are 55° and 68°. If the tracking stations are 20 km apart, determine the altitude of the weather balloon.

Marnie's Solution



I drew a diagram. I did not have enough information about $\triangle ADC$ to calculate h, but I did have enough information about $\triangle ACB$ to calculate x. I reasoned that if I could determine the hypotenuse, x, in the right triangle that contained the 55° angle, then I could use a primary trigonometric ratio to calculate the altitude, h, of the balloon.

$$\angle ACB = 180° - 55° - 68°$$
$$= 57°$$

I knew that the angles in $\triangle ACB$ added up to 180°.

$$\frac{x}{\sin 68°} = \frac{20}{\sin 57°}$$
$$x = \sin 68°\left(\frac{20}{\sin 57°}\right)$$
$$x \doteq 22.1$$

I knew $\angle ACB$ and the side opposite it in $\triangle ACB$. I also knew $\angle B$, which is the angle opposite side x.

I used the sine law to write a proportion that related these values. Then I solved for x.

$$\sin A = \frac{h}{x}$$
$$\sin 55° = \frac{h}{22.1}$$
$$(22.1)(\sin 55°) = h$$
$$18.1 \doteq h$$

$\triangle ACD$ is a right triangle in which h is opposite the 55° angle and x is the hypotenuse.

I used the sine ratio to write an equation. Then I solved for h.

The altitude of the weather balloon is about 18 km.

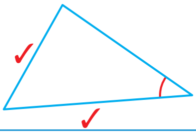
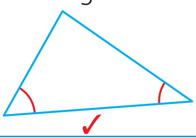
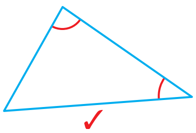
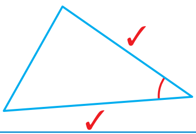
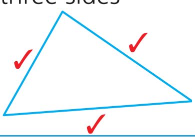
In Summary

Key Ideas

- If a real-world problem can be modelled using an acute triangle, the sine law or cosine law, sometimes along with the primary trigonometric ratios, can be used to determine unknown measurements.
- Drawing a clearly labelled diagram makes it easier to select a strategy for solving the problem.

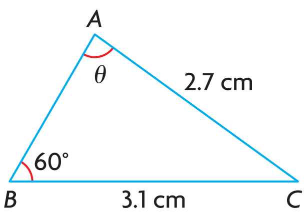
Need to Know

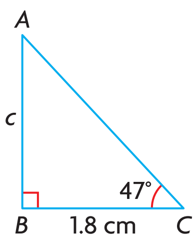
- To decide whether you need to use the sine law or the cosine law, consider the information given about the triangle and the measurement to be determined.

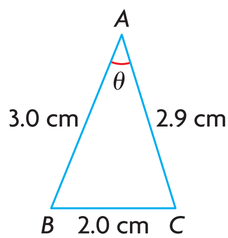
Information Given	Measurement To Be Determined	Use
two sides and the angle opposite one of the sides 	angle	sine law
two angles and a side  	side	sine law
two sides and the contained angle 	side	cosine law
three sides 	angle	cosine law

CHECK Your Understanding

1. Explain how you would determine the measurement of the indicated angle or side in each triangle.

a)


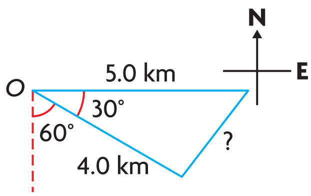
b)


c)


2. Use the strategies you described to determine the measurements of the indicated angles and sides in question 1.

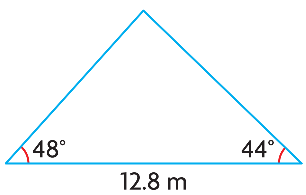
PRACTISING

3. The angle between two equal sides of an isosceles triangle is 52°. **K** Each of the equal sides is 18 cm long.
   a) Determine the measures of the two equal angles in the triangle.
   b) Determine the length of the third side.
   c) Determine the perimeter of the triangle.



4. A boat leaves Oakville and heads due east for 5.0 km as shown in the diagram at the left. At the same time, a second boat travels in a direction S60°E from Oakville for 4.0 km. How far apart are the boats when they reach their respective destinations?

5. A radar operator on a ship discovers a large sunken vessel lying flat on the ocean floor, 200 m directly below the ship. The radar operator measures the angles of depression to the front and back of the sunken ship to be 56° and 62°. How long is the sunken ship?



6. The base of a roof is 12.8 m wide as shown in the diagram at the left. The rafters form angles of 48° and 44° with the horizontal. How long, to the nearest tenth of a metre, is each rafter?

7. A flagpole stands on top of a building that is 27 m high. From a point on the ground some distance away, the angle of elevation to the top of the flagpole is 43°. The angle of elevation to the bottom of the flagpole is 32°.
   a) How far is the point on the ground from the base of the building?
   b) How tall is the flagpole?

8. Two ships, the *Albacore* and the *Bonito*, are 50 km apart. The *Albacore* **A** is N45°W of the *Bonito*. The *Albacore* sights a distress flare at S5°E. The *Bonito* sights the distress flare at S50°W. How far is each ship from the distress flare?

Career Connection

Pilots and flight engineers transport people, goods, and cargo. Some test aircraft, monitor air traffic, rescue people, or spread seeds for reforesting.

9. Fred and Agnes are 520 m apart. As Brendan flies overhead in an airplane, they measure the angle of elevation of the airplane. Fred measures the angle of elevation to be 63°. Agnes measures it to be 36°. What is the altitude of the airplane?

10. The *Nautilus* is sailing due east toward a buoy. At the same time, the *Porpoise* is approaching the buoy heading N42°E. If the *Nautilus* is 5.4 km from the buoy and the *Porpoise* is 4.0 km from the *Nautilus*, on a heading of S46°E, how far is the *Porpoise* from the buoy?

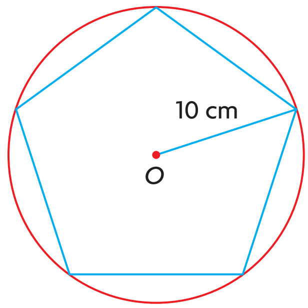
11. Two support wires are fastened to the top of a satellite dish tower from points A and B on the ground, on either side of the tower. One wire is 18 m long, and the other wire is 12 m long. The angle of elevation of the longer wire to the top of the tower is 38°.

a) How tall is the satellite dish tower?

b) How far apart are points A and B?

12. A regular pentagon is inscribed in a circle with radius 10 cm as shown in the diagram at the right. Determine the perimeter of the pentagon.

T



13. Ryan is in a police helicopter 400 m directly above a highway. When he looks west, the angle of depression to a car accident is 65°. When he looks east, the angle of depression to the approaching ambulance is 30°.

a) How far away is the ambulance from the scene of the accident?

b) The ambulance is travelling at 80 km/h. How long will it take the ambulance to reach the scene of the accident?

14. The radar screen in an air-traffic control tower shows that two airplanes are at the same altitude. According to the range finder, one airplane is 100 km away, in the direction N60°E. The other airplane is 160 km away, in the direction S50°E.

a) How far apart are the airplanes?

b) If the airplanes are approaching the airport at the same speed, which airplane will arrive first?

15. In a parallelogram, two adjacent sides measure 10 cm and 12 cm. The shorter diagonal is 15 cm. Determine, to the nearest degree, the measures of all four angles in the parallelogram.

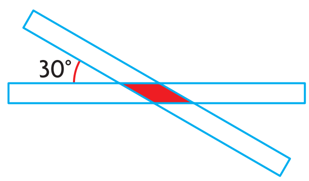
16. Create a real-life problem that can be modelled by an acute triangle. Then describe the problem, sketch the situation in your problem, and explain what must be done to solve it.

C

Extending

17. From the top of a bridge that is 50 m high, two boats can be seen anchored in a marina. One boat is anchored in the direction S20°W, and its angle of depression is 40°. The other boat is anchored in the direction S60°E, and its angle of depression is 30°. Determine the distance between the two boats.

18. Two paper strips, each 5 cm wide, are laid across each other at an angle of 30°, as shown at the right. Determine the area of the overlapping region. Round your answer to the nearest tenth of a square centimetre.



8 Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

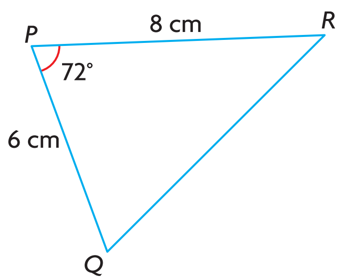
- See Lesson 8.3 and Lesson 8.4, Examples 1 to 3.
- Try Chapter Review Questions 8 to 10.

Q: To use the cosine law, what do you need to know about a triangle?

A: You need to know the measurements of three sides, or two sides and the contained angle in the triangle. You can calculate the length of a side if you know the measure of the angle that is opposite the side, as well as the lengths of the other two sides. You can calculate the measure of an angle if you know the lengths of all three sides.

EXAMPLE

Can you use the cosine law to determine the length of *RQ*? Explain.



Solution

Yes. You know the lengths of two sides and the angle that is opposite the side you want to determine. Solving for p will give you the measure of *RQ*.

$p^2 = 8^2 + 6^2 - 2(8)(6)\cos 72°$

Study *Aid*

- See Lesson 8.5, Examples 1 to 3.
- Try Chapter Review Questions 11 to 13.

Q: When solving a problem that can be modelled by an acute triangle, how do you decide whether to use the primary trigonometric ratios, the sine law, or the cosine law?

A: Draw a clearly labelled diagram of the situation to see what you know.

- If the diagram involves one or more right triangles, you might be able to use a primary trigonometric ratio.
- Use the sine law if you know the lengths of two sides and the measure of one opposite angle, or the measures of two angles and the length of one opposite side.
- Use the cosine law if you know the lengths of all three sides, or two sides and the angle between them.

You may need to use more than one strategy to solve some problems.

PRACTICE Questions

Lesson 8.1

1. Jane claims that she can draw an acute triangle using the following information: $a = 6$ cm, $b = 8$ cm, $c = 10$ cm, $\angle A = 30°$, and $\angle B = 60°$. Is she correct? Explain.

2. Which of the following are not correct for acute triangle *DEF*?

a) $\dfrac{d}{\sin D} = \dfrac{f}{\sin F}$

b) $\dfrac{\sin E}{e} = \dfrac{\sin D}{d}$

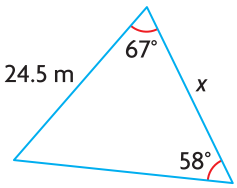
c) $f \sin E = e \sin F$

d) $\dfrac{d}{\sin D} = \dfrac{\sin F}{f}$

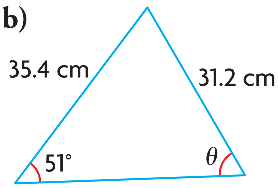
Lesson 8.2

3. Calculate the indicated side length or angle measure in each triangle.

a)



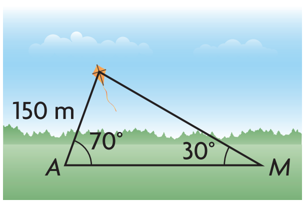
b)



4. In $\triangle ABC$, $\angle B = 31°$, $b = 22$ cm, and $c = 12$ cm. Determine $\angle C$.

5. Solve $\triangle ABC$, if $\angle A = 75°$, $\angle B = 50°$, and the side between these angles is 8.0 cm.

6. Allison is flying a kite. She has released the entire 150 m ball of kite string. She notices that the string forms a 70° angle with the ground. Marc is on the other side of the kite and sights the kite at an angle of elevation of 30°. How far is Marc from Allison?



Lesson 8.3

7. Which of these is not a form of the cosine law for $\triangle ABC$? Why not?

a) $a^2 = b^2 + c^2 - 2\,bc \cos B$

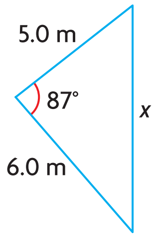
b) $c^2 = a^2 + b^2 - 2\,ab \cos C$

c) $b^2 = a^2 + c^2 - 2\,ac \cos B$

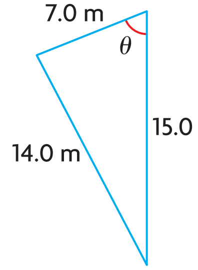
Lesson 8.4

8. Calculate the indicated side length or angle measure.

a)



b)

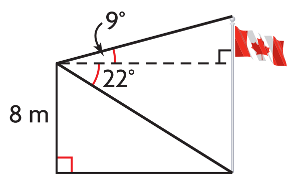


9. Solve $\triangle ABC$, if $\angle A = 58°$, $b = 10.0$ cm, and $c = 14.0$ cm.

10. Two airplanes leave an airport at the same time. One airplane travels at 355 km/h. The other airplane travels at 450 km/h. About 2 h later, they are 800 km apart. Determine the angle between their paths.

Lesson 8.5

11. From the top of an 8 m house, the angle of elevation to the top of a flagpole across the street is 9°. The angle of depression is 22° to the base of the flagpole. How tall is the flagpole?



12. A bush pilot delivers supplies to a remote camp by flying 255 km in the direction N52°E. While at the camp, the pilot receives a radio message to pick up a passenger at a village. The village is 85 km S21°E from the camp. What is the total distance that the pilot will have flown by the time he returns to his starting point?

13. A canoeist starts from a dock and paddles 2.8 km N34°E. Then she paddles 5.2 km N65°W. What distance, and in which direction, should a second canoeist paddle to reach the same location directly, starting from the same dock?

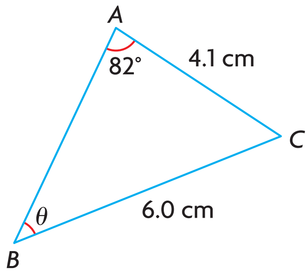
8 Chapter Self-Test

Process Checklist

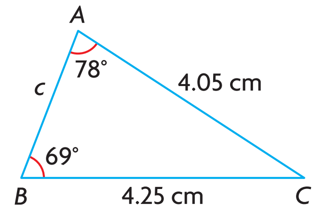
- ✔ Questions 1, 5, and 7: Did you use the given **representations** to help you select an appropriate strategy?
- ✔ Question 3: Did you **reflect** on the information needed to use the sine law as you sketched and labelled the triangle?
- ✔ Questions 4, 6, and 8: Did you make **connections** between each situation and acute triangle trigonometry?
- ✔ Question 9: Did you **communicate** your thinking clearly?

1. Determine the indicated side length or angle measure in each triangle.

a)



b)

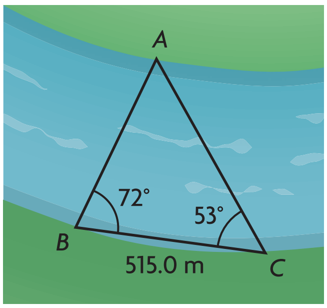


2. In $\triangle PQR$, $\angle P = 80°$, $\angle Q = 48°$, and $r = 20$ cm. Solve $\triangle PQR$.

3. a) Sketch an acute triangle. Label three pieces of information (side lengths or angle measures) so that the sine law can be used to determine at least one of the unknown side lengths or angle measures.

b) Use the sine law to determine one unknown side length or angle measure in your triangle.

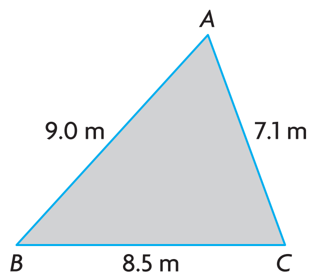
4. The radar screen of a Coast Guard rescue ship shows that two boats are in the area. According to the range finder, one boat is 70 km away, in the direction N45°E. The other boat is 100 km away, in the direction S50°E. How far apart are the two boats?



5. An engineer wants to build a bridge over a river from point A to point B as shown in the diagram at the left. The distance from point B to point C is 515.0 m. The engineer uses a transit to determine that $\angle B$ is 72° and $\angle C$ is 53°. Determine the length of the finished bridge.

6. A parallelogram has adjacent sides that are 11.0 cm and 15.0 cm long. The angle between these sides is 50°. Determine the length of the shorter diagonal.

7. Terry is designing a new triangular patio. The diagram at the right shows the dimensions of the patio. Calculate the area of the patio.



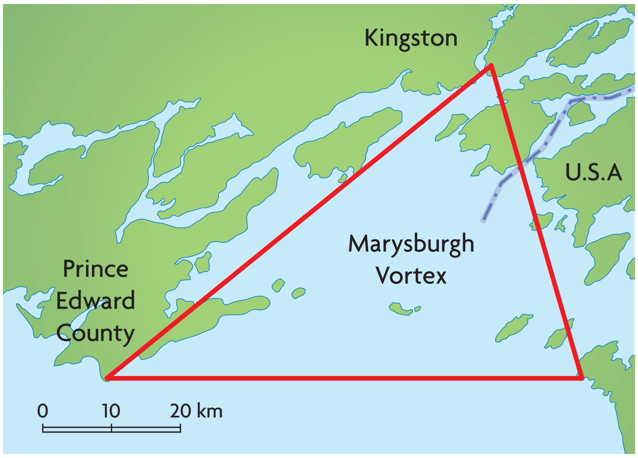
8. Points P and Q lie 240 m apart on opposite sides of a communications tower. The angles of elevation to the top of the tower from P and Q are 50° and 45°, respectively. Calculate the height of the tower.

9. In an acute triangle, two sides are 2.4 cm and 3.6 cm. One of the angles is 37°. How can you determine the third side in the triangle? Explain.

8 Chapter Task

Dangerous Triangles

Some people claim that there is a region in the eastern end of Lake Ontario near Kingston, called the Marysburgh Vortex, where more than two-thirds of the shipwrecks in the lake are found. They say that the Marysburgh Vortex is similar to the Bermuda Triangle because of its strange habit of swallowing boats and airplanes. There are estimates of up to 450 wrecks in the Marysburgh Vortex, with about 80 of them in the area from Kingston to Prince Edward County. There are estimates of about 1000 wrecks in the Bermuda Triangle.



Is the Marysburgh Vortex more dangerous than the Bermuda Triangle?

A. Use the map to estimate the lengths of the sides that form the triangle of the Marysburgh Vortex.

B. Use a strategy that does not involve trigonometry to determine the area of the triangle for part A.

C. Use trigonometry to calculate the area of the triangle for part A.

D. Compare your answers for parts B and C.

E. Calculate the area of the Bermuda Triangle.

F. Which region do you think is more dangerous for sailing? Justify your decision.

Task *Checklist*

- ✔ Did you draw labelled diagrams for the problem?
- ✔ Did you show your work?
- ✔ Did you provide appropriate reasoning?
- ✔ Did you explain your thinking clearly?